MA114 Summer 2018
Worksheet 6 – Improper Integrals – 6/15/18

1. Compute the following integrals.

(a) $\displaystyle\int_1^\infty \frac{dx}{x^{19/20}} = \lim_{R\to\infty} \int_1^R \frac{dx}{x^{19/20}} = \lim_{R\to\infty} 20x^{\frac{1}{20}} \Big|_1^R$

$= \lim_{R\to\infty} 20\left(R^{\frac{1}{20}} - 1\right)$

$= \infty$

<u>Divergent</u>

(b) $\displaystyle\int_1^\infty \frac{dx}{x^{20/19}} = \lim_{R\to\infty} \int_1^R \frac{1}{x^{20/19}}\,dx = \lim_{R\to\infty} -19\,x^{-\frac{1}{19}} \Big|_1^R$

$= \lim_{R\to\infty} \frac{-19}{R^{1/19}} + 19$

$= 0 + 19$

$= \boxed{19}$

$\cancel{u = .000001t \quad du = .000001\,dt \Rightarrow dt =}$

(c) $\displaystyle\int_{-\infty}^4 e^{0.000001t}\,dt. = \lim_{R\to-\infty} \int_R^4 e^{10^{-6}t}\,dt = \lim_{R\to-\infty} 10^6 e^{10^{-6}t} \Big|_R^4$

$= \lim_{R\to-\infty} 10^6\left(e^{4\cdot10^{-6}} - e^{10^{-6}R}\right)$

$= 10^6\left(e^{4\cdot10^{-6}} - 0\right)$

$\approx 1{,}000{,}004$

2. Consider

$$\int_1^\infty \frac{dx}{x^p}.$$

For what values of p does the integral converge? For what values does it diverge? Justify your answer. (Think about 1a, 1b, and the examples from lecture.)

Guess from examples: converge if $p>1$, diverge if $p \le 1$.

$p=1$: $\int_1^\infty \frac{1}{x}\,dx$ done on Friday.

Otherwise, if $p \neq 1$: $\int_1^\infty \frac{dx}{x^p} = \lim_{R\to\infty} \int_1^R \frac{1}{x^p}\,dx = \lim_{R\to\infty} \frac{1}{1-p} x^{1-p}\Big|_1^R = \lim_{R\to\infty} \frac{1}{1-p}(R^{1-p}-1)$

$$= \begin{cases} \infty, & \text{if } 1-p>0 \text{ i.e. } p<1 \\ \frac{1}{p-1}, & \text{if } 1-p<0 \text{ i.e. } p>1 \end{cases}$$

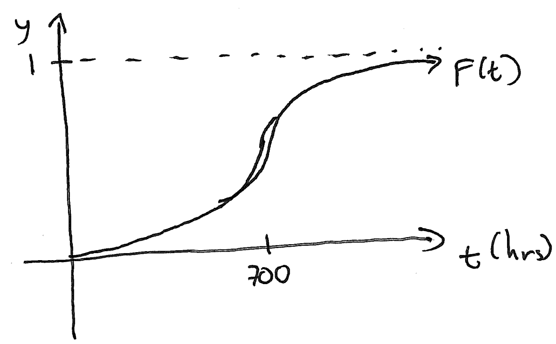
3. A manufacturer of lightbulbs wants to produce bulbs that last about 700 hours but, of course, some bulbs burn out faster than others. Let $F(t)$ be the fraction of the company's bulbs that burn out before t hours, so $F(t)$ always lies between 0 and 1.

   (a) Make a rough sketch of what you think the graph of $F(t)$ might look like.

   (b) What is the meaning of the derivative $r(t) = F'(t)$?

   (c) What is the value of $\int_0^\infty r(t)dt$? Why?

a) We should expect few bulbs to burn out near 0 hours, about half to burn out ~~around~~ by 700 hours, and as time goes on all bulbs to burn out:



b) $r(t)$ is the fractional rate of burnout of bulbs

c) $\int_0^\infty r(t)dt = 1$, because $\int_0^\infty F'(t)\,dt$

$= \lim_{R\to\infty} \int_0^R F'(t)dt = \lim_{R\to\infty} F(R) - \cancel{F(0)}^{\,0}$

$= 1$

since we expect all bulbs to eventually burn out.